16) **วิธีทำ** จากโจทย์ให้หา $\sin(180^\circ + \arctan x)$

จากสูตรคำนวณ $\quad \sin(A+B) = \sin A\cos B + \cos A\sin B$

ดังนั้น $\quad \sin(180^\circ + \arctan x) = \sin 180^\circ\cos(\arctan x) + \cos 180^\circ\sin(\arctan x)$

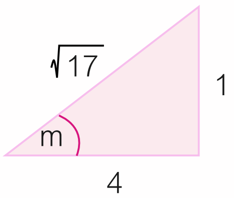
แทนค่า $\sin 180^\circ = 0$ และ $\cos 180^\circ = -1$ ลงในสมการ

$$\sin(180^\circ + \arctan x) = (0)\cos(\arctan x) + (-1)\sin(\arctan x)$$

$$= -1\sin(\arctan x)$$

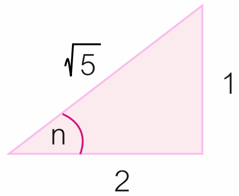
$$= -\sin(\arctan x)$$

กำหนดให้ $\arctan\frac{1}{4} = m$ และ $\arctan\frac{1}{2} = n$



$$\sin m = \frac{1}{\sqrt{17}}$$

$$\cos m = \frac{4}{\sqrt{17}}$$



$$\sin n = \frac{1}{\sqrt{5}}$$

$$\cos n = \frac{2}{\sqrt{5}}$$

จากโจทย์ $\arctan x = \arctan\frac{1}{4} - 2\arctan\frac{1}{2}$

ดังนั้น $\quad \arctan x = m - 2n$

ใส่ sin ทั้ง 2 ฝั่งของสมการ ; $\quad \sin(\arctan x) = \sin(m-2n)$

จากสูตร $\quad \sin(A-B) = \sin A\cos B - \cos A\sin B$

$$\sin(m-2n) = \sin m\cos 2n - \cos m\sin 2n$$

จากสูตร $\quad \cos(2\theta) = 2\cos^2\theta - 1$ และ $\sin(2\theta) = 2\sin\theta\cos\theta$

$$\sin(m-2n) = (\sin m)(2\cos^2 n - 1) - (\cos m)[2(\sin n)(\cos n)]$$

$$= \left(\frac{1}{\sqrt{17}}\right)\left[2\left(\frac{2}{\sqrt{5}}\right)^2 - 1\right] - \left(\frac{4}{\sqrt{17}}\right)(2)\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$= \left(\frac{1}{\sqrt{17}}\right)\left[2\left(\frac{4}{5}\right) - 1\right] - \left(\frac{4}{\sqrt{17}}\right)(2)\left(\frac{2}{5}\right)$$

$$= \left(\frac{1}{\sqrt{17}}\right)\left(\frac{8-5}{5}\right) - \left(\frac{4}{\sqrt{17}}\right)\left(\frac{4}{5}\right)$$

$$= \left(\frac{1}{\sqrt{17}}\right)\left(\frac{3}{5}\right) - \left(\frac{1}{\sqrt{17}}\right)\left(\frac{16}{5}\right)$$

ดังนั้น $\quad \sin(\arctan x) = \frac{-13}{5\sqrt{17}}$

$$\therefore \quad \sin(180^\circ + \arctan x) = -\sin(\arctan x) = -\left(\frac{-13}{5\sqrt{17}}\right) = \frac{13}{5\sqrt{17}}$$

ตอบ C